Published by Institute of Physics Publishing for SISSA/ISAS
Received: *February 22, 2002*
Revised: *March 07, 2002*
Accepted: *March 07, 2002*

Tachyons, supertubes and brane/anti-brane systems

David Mateos, Selena Ng and Paul K. Townsend

Department of Applied Mathematics and Theoretical Physics
Centre for Mathematical Sciences
Wilberforce Road, Cambridge CB3 0WA, UK
E-mail: D.Mateos@damtp.cam.ac.uk, S.K.L.Ng@damtp.cam.ac.uk, P.K.Townsend@damtp.cam.ac.uk

Abstract: We find supertubes with *arbitrary* (and not necessarily planar) cross-section; the stability against the D2-brane tension is due to a compensation by the local momentum generated by Born-Infeld fields. Stability against long-range supergravity forces is also established. We find the corresponding solutions of the $N = \infty$ M(atrix) model. The supersymmetric $D2/\overline{D2}$ system is a special case of the general supertube, and we show that there are no open-string tachyons in this system via a computation of the open-string one-loop vacuum energy.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Supersymmetry and Duality.

Contents

1. Introduction

A collection of branes may (under some circumstances) find it energetically favourable to expand to form a brane of higher dimension [1, 2]. This "brane expansion" plays a fundamental role in a number of phenomena, such as the behaviour of gravitons at high energies in certain backgrounds [3] or the string realization of the vacuum structure of $\mathcal{N} = 1$ four-dimensional gauge theories [4]. The presence of the new brane "created" by the expansion is implied by the fact that the expanded system couples locally to higher-rank gauge fields under which the original constituents are neutral.[1] For a large number of constituent branes there may then be an effective description in terms of the higher-dimensional brane in which the original branes have become fluxes of various types. Since no *net* higher-dimensional brane charge is created by the process of expansion, at least one of the dimensions of the higher-dimensional brane must be compact and homologically trivial.

Although the phenomenon of brane expansion was originally discovered in the context of supersymmetric theories, early examples of "expanded-brane" configurations were not themselves supersymmetric, and hence unlikely to be stable. Supersymmetric expanded

[1] This feature is absent from a phenomenon such as the enhançon [5] which some authors also refer to as "brane expansion".

brane configurations in certain backgrounds have been found[2] but the presence of non-vanishing Ramond-Ramond fields in all of them makes any conventional string theory analysis difficult. At present, the only example of a supersymmetric expanded brane configuration in a vacuum background is the D2-brane "supertube" of [6], which was provided with a Matrix Theory interpretation in [7]. Supertubes are collections of type-IIA fundamental strings and D0-branes which have been expanded, in the IIA Minkowski vacuum, to tubular 1/4-supersymmetric D2-branes by the addition of angular momentum.[3]

The simplest potential instability of any expanded brane configuration is that caused by brane tension, which tends to force the system to contract. The presence or absence of this instability is captured by the low-energy effective action for the brane in question, e.g. the Dirac-Born-Infeld (DBI) action for D-branes. The D2-brane DBI action is what was used in [6] to find the D2-brane supertube (with circular cross-section) and to establish its supersymmetry. Although the original supertube was assumed to have a circular cross-section, it was shown in [9], in the Matrix Theory context, that an elliptical cross-section is also possible. One purpose of this paper is to generalize the results of [6] to show that 1/4-supersymmetric supertubes may have as a cross-section a completely *arbitrary* curve in $\mathbb{E}^8$. The stability of the original supertube was attributed to the angular momentum generated by the Born-Infeld (BI) fields. Although this is still true, the stability of the general supertube configurations is less easily understood; as we shall see the D2-brane tension is not isotropic and the supertube behaves in some respects like a *tensionless* brane. We also find these general supertube configurations as solutions of the light-front gauge-fixed eleven-dimensional supermembrane equations. As this is the $N = \infty$ limit of the Matrix model, we thus make contact with the Matrix Theory approach of [7, 9].

A second potential instability is that associated to the long-range forces between different regions of an expanded brane (or between different expanded branes) due to the exchange of massless particles. In the case of the D2-brane supertube these are particles in the closed IIA superstring spectrum. Whether or not this instability occurs may be determined by considering the D2-brane supertube in the context of the effective IIA supergravity theory. For circular supertubes this aspect was considered in [10], where a 1/4-supersymmetric supergravity solution for a general multi-supertube system was constructed and stability against supergravity forces confirmed.[4] Here we shall generalize these results to supertubes with arbitrary cross-section by explicitly exhibiting the corresponding supergravity solutions.

In the case of D-branes (at least) there is a third more dramatic and purely "stringy" potential instability of an expanded brane configuration arising from the fact that opposite sides (along one of the compact directions) of the (higher-dimensional) D-brane behave locally as a brane/anti-brane pair. For sufficiently small separation there are tachyons in the open strings between a D-brane and an anti-D-brane, so a sufficiently compact

[2]For example (but not exclusively) in AdS backgrounds [3, 4]. We thank Iosif Bena for correspondence on this point.

[3]*Non*-supersymmetric brane expansion by angular momentum in Minkowski space was considered previously in [8].

[4]Multi-supertube systems have been considered in the context of Matrix Theory in [7].

expanded D-brane configuration is potentially unstable against tachyon condensation [11]. One purpose of this paper is to examine this issue for the D2-brane supertube. One would certainly expect it to be stable given that it preserves supersymmetry, but supersymmetry has been established only within the context of the effective DBI action and the effective supergravity theory. Thus the preservation of supersymmetry could be an artifact of these effective theories that is not reproduced within the full string theory; the stability of the supertube in the full IIA string theory is therefore not guaranteed *a priori*.

The main obstacle to a full string theory analysis of the supertube is that, as for any other expanded brane configuration, the D-brane surface must be (extrinsically) curved, so even though we have the advantage of a vacuum spacetime background the quantization of open strings attached to the D2-brane supertube is a difficult problem. However, it seems reasonable to suppose that any tachyons in the supertube system would also appear in the case in which the D2-brane is locally approximated by a flat "tangent-brane". In fact, because the cross-section of the supertube is *arbitrary* we may take a limiting case in which the tube becomes a pair of D2-branes which intersect at an arbitrary angle ϕ; the particular case $\phi = \pi$ corresponds to a parallel D2/anti-D2 pair, discussed in [9] in the Matrix model context. Another purpose of this paper is to show that there is no tachyon in this system for any angle. We establish this by computing the one-loop vacuum energy of the open strings stretched between the D2-branes.

2. Supertubes with arbitrary cross-sections

In the original paper on supertubes [6] only circular cross-sections were considered. This was later generalized in the context of Matrix theory [9] to elliptical cross-sections. The purpose of this section is to show that any tubular D2-brane with an arbitrary cross-section (not necessarily contained in a 2-plane nor closed) carrying the appropriate string and D0-brane charges is also 1/4-supersymmetric. This point is not immediately obvious from the approach in [9] and it was missed in the DBI analysis of [6].

2.1 Supersymmetry

Consider a D2-brane with worldvolume coordinates $\xi^a = \{t, z, \sigma\}$ in the type-IIA Minkowski vacuum. We write the spacetime metric as

$$ds^2_{10} = -dT^2 + dZ^2 + d\vec{Y} \cdot d\vec{Y} \,, \tag{2.1}$$

where $\vec{Y} = \{Y^i\}$ are cartesian coordinates on $\mathbb{E}^8$, and set

$$T = t \,, \qquad Z = z \,, \qquad \vec{Y} = \vec{y}(\sigma) \,. \tag{2.2}$$

This describes a static tubular D2-brane whose axis is aligned with the Z-direction and whose cross-section is an arbitrary curve $\vec{y}(\sigma)$ in $\mathbb{E}^8$ (see figure 1). Although the term "tubular" is strictly appropriate only when the curve is closed, we will use it to refer to any of these configurations.

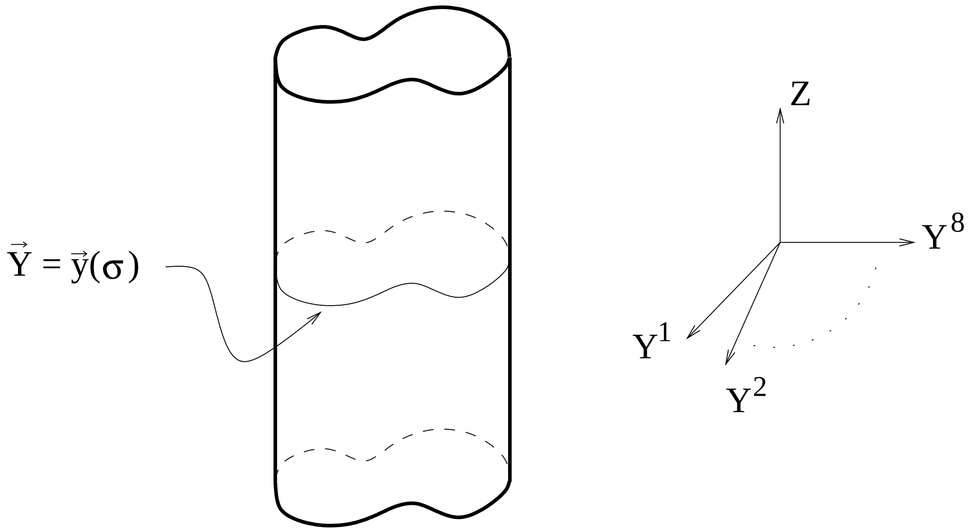

Figure 1: Supertube with arbitrary cross-section $\vec{Y} = \vec{y}(\sigma)$ in $\mathbb{E}^8$.

We will also allow for time- and z-independent electric (in the Z-direction) and magnetic fields, so the BI field-strength on the D2-brane is

$$F = E\, dt \wedge dz + B(\sigma)\, dz \wedge d\sigma\,. \tag{2.3}$$

As we will see in more detail below, this corresponds to having string charge in the Z-direction and D0-brane charge dissolved in the D2-brane. Note that, under the assumption of time- and z-independence, closure of F implies that E must be constant but still allows B to depend on σ.

The supersymmetries of the IIA Minkowski vacuum preserved by a D2-brane configuration are those generated by (constant) Killing spinors ϵ satisfying $\Gamma\epsilon = \epsilon$, where Γ is the matrix appearing in the "kappa-symmetry" transformation of the D2-brane worldvolume spinors [12]. For the configuration of interest here this condition reduces to

$$y'_i\, \Gamma_i\, \Gamma_\natural\, \left(\Gamma_{TZ}\Gamma_\natural + E\right) \epsilon + \left(B\, \Gamma_T \Gamma_\natural - \sqrt{(1-E^2)|\vec{y}\,'|^2 + B^2}\right) \epsilon = 0\,, \tag{2.4}$$

where Γ_T, Γ_Z and Γ_i are (constant) ten-dimensional Minkowski spacetime Dirac matrices, $\Gamma_\natural = \Gamma_T \Gamma_Z \Gamma_1 \cdots \Gamma_8$ is the chirality matrix in ten dimensions, and the prime denotes differentiation with respect to σ.

The supersymmetry preservation equation (2.4) is satisfied for any arbitrary curve provided that we set $|E| = 1$, impose the two conditions

$$\Gamma_{TZ}\Gamma_\natural\, \epsilon = -\,\mathrm{sgn}(E)\, \epsilon\,, \qquad \Gamma_T \Gamma_\natural\, \epsilon = \mathrm{sgn}(B)\, \epsilon \tag{2.5}$$

on the Killing spinors, and demand that $B(\sigma)$ is a constant-sign, but otherwise completely arbitrary, function of σ. We would like to emphasize that $|E| = 1$ is *not* a critical electric field in the presence of a nowhere-vanishing magnetic field $B(\sigma)$, which is the only case that we will consider in this paper.

The two conditions (2.5) on ϵ correspond to string charge along the Z-direction and to D0-brane charge, respectively, and preserve 1/4 of the supersymmetry.[5] Note that the D2-brane projector does not appear; we shall return to this issue later. It is straightforward to check that this configuration satisfies the equations of motion derived from the D2-brane action, which (for unit surface tension) is

$$S_{D2} = \int dt\, dz\, d\sigma\, \mathcal{L}_{D2} = -\int dt\, dz\, d\sigma\, \sqrt{-\det(g+F)}\,, \tag{2.6}$$

where

$$ds^2(g) = -dt^2 + dz^2 + |\vec{y}\,'|^2 d\sigma^2 \tag{2.7}$$

is the induced metric on the D2-brane worldvolume. Note that the area element is

$$\sqrt{\det \mathrm{g}_{\mathrm{spatial}}} = |\vec{y}\,'|\,. \tag{2.8}$$

2.2 Mechanical stability: momentum versus tension

We have concluded that a D2-brane with an arbitrary shape and an arbitrary magnetic field $B(\sigma)$ preserves 1/4 of the supersymmetries of the IIA vacuum, and therefore must be stable. In "normal" circumstances this would be impossible because of the D2-brane tension. In the present case this tension is exactly balanced by the "centrifugal" force associated to the linear momentum density carried by the supertube, as we show below. The origin of this momentum is not time-dependence (which would generically be incompatible with supersymmetry) but the Poynting vector generated by the crossed static electric and magnetic fields on the D2-brane.

The linear momentum density in the i-direction is easily computed by momentarily allowing Y^i to depend on time, evaluating

$$\mathcal{P}_i = \frac{\partial \mathcal{L}_{D2}}{\partial \dot{Y}^i}\,, \tag{2.9}$$

where the dot denotes differentiation with respect to time, and finally setting $\dot{Y}^i = 0$. The result is

$$\mathcal{P}_i = \frac{B E y_i'}{\sqrt{(1-E^2)|\vec{y}\,'|^2 + B^2}}\,. \tag{2.10}$$

Similarly, the momentum conjugate to the electric field or "electric displacement" is

$$\Pi(\sigma) = \frac{\partial \mathcal{L}_{D2}}{\partial E} = \frac{E|\vec{y}\,'|^2}{\sqrt{(1-E^2)|\vec{y}\,'|^2 + B^2}}\,. \tag{2.11}$$

For supersymmetric configurations $|E| = 1$ and the momenta (2.10) become

$$\mathcal{P}_i = \mathrm{sgn}(\Pi B)\, y_i'\,. \tag{2.12}$$

[5] Equation (2.4) admits other types of solutions for particular forms of the cross-section. For example, if this is a straight line and B is constant, then the fraction of preserved supersymmetry is 1/2, corresponding to an infinite planar D2-brane with a homogeneous density of bound strings and D0-branes. In this paper we will concentrate on the generic case of an arbitrary cross-section.

JHEP03(2002)016

This combined with (2.11) yields the condition

$$|\vec{\mathcal{P}}|^2 = |\Pi B|\,. \tag{2.13}$$

Note that the densities $\Pi(\sigma)$, $B(\sigma)$ and $\mathcal{P}_i(\sigma)$ depend on the parametrization of the curve $\vec{y}(\sigma)$. However, the physical densities per unit D2-brane area are reparametrization-invariant. Using (2.8) these are

$$\Pi_{\rm ph}(\sigma) = \frac{\Pi(\sigma)}{|\vec{y}\,'|}\,, \qquad \mathrm{B}_{\rm ph}(\sigma) = \frac{B(\sigma)}{|\vec{y}\,'|}\,, \qquad \mathcal{P}^i_{\rm ph}(\sigma) = \frac{\mathcal{P}^i(\sigma)}{|\vec{y}\,'|}\,. \tag{2.14}$$

Note that, by virtue of (2.12), the momentum density per unit area has constant unit magnitude, that is, $|\vec{\mathcal{P}}_{\rm ph}| = 1$.

The stability of the supertube with an arbitrary shape may now be understood in "mechanical" terms as follows. The conserved string charge and the D0-brane charge per unit length in the Z-direction carried by the supertube are (for an appropriate choice of units)

$$q_{F1} = \int d\sigma\, \Pi\,, \qquad q_{D0} = \int d\sigma\, B\,. \tag{2.15}$$

The string and D0-brane charge densities $\Pi(\sigma)$ and $B(\sigma)$ generate a Poynting linear momentum density $\vec{\mathcal{P}}$ at each point along the cross-section. For specified shape and magnetic field B, supersymmetry automatically adjusts Π in such a way that the momentum density per unit area is tangent to the curve and has constant magnitude. The fact that it is tangent means that, in a mechanical analogy, one can think of this momentum as originating from a continuous motion of the curve along itself, similarly to that of a fluid along a tube of fixed shape. The fact that it is constant in magnitude means that the force acting on each point of the curve must be orthogonal to the momentum: this is precisely the force due to the D2-brane tension. In the absence of momentum, this tension would make the curve collapse, whereas here it provides precisely the required centripetal force to direct the motion of each point on the curve in such a way that the curve is mapped into itself under this motion. This is how stability is achieved for an arbitrary shape.

One consequence of the precise balance of forces discussed above is that the D2-brane behaves in a certain sense as a *tensionless* brane. To see this, it is important to remember that the tension of a system is *not* the same as its energy per unit volume, and also that it may be non-isotropic; the supertube is an example of this. The tension tensor is defined as minus the purely spatial part of the spacetime stress-energy tensor. The latter is computed as

$$T^{MN}(x) = \frac{2}{\sqrt{-\det G}}\, \frac{\delta S_{D2}}{\delta G_{MN}(x)}\bigg|_{G_{MN}=\eta_{MN}}\,, \qquad (M,N = 0,\ldots,9) \tag{2.16}$$

by momentarily allowing for a general spacetime metric in the D2-brane action and then setting it to its Minkowski value (2.1) after evaluating the variation above. The result is

$$T^{MN}(x) = \int d^3\xi\, \mathcal{T}^{MN}(X(\xi))\, \delta^{(10)}(x - X(\xi))\,, \tag{2.17}$$

where

$$\mathcal{T}^{MN} = -\sqrt{-\det(g+F)}\ \left[(g+F)^{-1}\right]^{(ab)} \partial_a X^M \partial_b X^N \tag{2.18}$$

and $X^M = \{T, Z, Y^i\}$. Note that $T^{MN}(x)$ is conserved, that is, $\partial_M T^{MN} = 0$, by virtue of the D2-brane equations of motion

$$\partial_a \left(\sqrt{\det(g+F)}\ \left[(g+F)^{-1}\right]^{ab} \partial_b X^M \right) = 0\,. \tag{2.19}$$

Specializing to the supertube configurations of interest here we find that the only non-zero components of $\mathcal{T}^{MN}$ are

$$\mathcal{T}^{TT} = |\Pi| + |B|\,, \qquad \mathcal{T}^{ZZ} = -|\Pi|\,, \qquad \mathcal{T}^{Ti} = \mathrm{sgn}(\Pi B)\, y_i'\,. \tag{2.20}$$

This result illustrates a number of points. First, the off-diagonal components $\mathcal{T}^{Ti}$ are precisely the linear momentum density (2.12) carried by the tube, as expected. Second, the fact that $\mathcal{T}^{ij}$ vanishes identically means that there is no tension along the cross-section; this provides a more formal explanation of why an arbitrary shape is stable. Third, the tube tension $-\mathcal{T}^{ZZ} = |\Pi|$ in the Z-direction is only due to the string density. Hence the D2-brane does not contribute to the tension in any direction: it has effectively become tensionless. (The fact that the D0-branes do not contribute to the tension either should be expected: they behave like dust.) Finally, the net energy of the supertube per unit length in the Z-direction

$$\mathcal{H} = \int d\sigma\, \mathcal{T}^{TT} = |q_{F1}| + |q_{D0}| \tag{2.21}$$

saturates the lower bound which we will derive from the supersymmetry algebra below. In particular, it receives contributions from the strings and the D0-branes, but not from the D2-brane. The reason for this is that the supertube is a true bound state in which the strictly negative energy which binds the strings and D0-branes to the D2-brane is exactly cancelled by the strictly positive energies associated to the mass of the D2-brane and to the presence of a linear momentum density.

2.3 Central charges

Supersymmetric configurations in a given theory are those which minimize the energy for fixed values of the central charges in the supersymmetry algebra of the theory. Both this energy and the precise set of preserved supersymmetries are completely determined once the central charges are specified. For the type-IIA theory, the anti-commutator of two supercharges in the presence of the central charges $\mathcal{Z}$ of interest here is

$$\{Q, Q\} = \Gamma^T \Gamma^M P_M + \frac{1}{2} \Gamma^T \Gamma^{MN} \mathcal{Z}^{D2}_{MN} + \Gamma^T \Gamma^M \Gamma_\natural \mathcal{Z}^{F1}_M + \Gamma^T \Gamma_\natural \mathcal{Z}^{D0}\,. \tag{2.22}$$

In the previous sections we have understood the supersymmetries preserved by the supertube and its mechanical stability from a local analysis. One purpose of this section is to show that this also follows from consideration of the central charges in the algebra above carried by the supertube. We shall also prove the existence of upper bounds on the total angular momentum and on the linear momentum density.

By definition, any supertube carries non-zero string and D0-brane charges. In addition, it may carry a total (per unit length in the Z-direction) linear momentum P_i and/or angular momentum 2-form $L_{ij} = -L_{ji}$ in $\mathbb{E}^8$. These are obtained by integrating the corresponding densities along the cross-section. For N overlapping D2-branes the linear momentum density (2.12) for supersymmetric configurations becomes

$$\mathcal{P}_i = N\, y_i' \,, \tag{2.23}$$

(where we have chosen $\mathrm{sgn}(\Pi B) = +1$ for concreteness) and hence the total linear momentum is

$$P_i = N \int d\sigma\, y_i' = N \int dy_i \,. \tag{2.24}$$

Note that the momentum density per unit area has magnitude N, that is,

$$|\vec{\mathcal{P}}_{\mathrm{ph}}| = N \,. \tag{2.25}$$

Similarly, the angular momentum density is

$$\mathcal{L}_{ij} \equiv Y_i\, \mathcal{P}_j - Y_j\, \mathcal{P}_i \,, \tag{2.26}$$

so in the supersymmetric case the total angular momentum takes the form

$$L_{ij} = N \int d\sigma\, \left(y_i\, y_j' - y_j\, y_i'\right) = N \int \left(y_i\, dy_j - y_j\, dy_i\right) . \tag{2.27}$$

Consider now a supertube with a closed cross-section. In this case the angular momentum in a given ij-plane is precisely the number of D2-branes times twice the area of the region $\mathcal{A}$ enclosed by the projection $\mathcal{C}$ of the cross-section onto that plane, since by Stoke's theorem we have

$$L_{ij} = N \oint_{\mathcal{C}} \left(y_i\, dy_j - y_j\, dy_i\right) = 2N \int_{\mathcal{A}} dy_i \wedge dy_j \,. \tag{2.28}$$

Consequently, a non-zero angular momentum prevents a closed cross-section from collapse, since the area enclosed by a collapsed cross-section would vanish. A closed supertube is therefore stabilized by the angular momentum. It may appear surprising that this can be done *supersymmetrically* in the Minkowski vacuum, since angular momentum is not a central charge in Minkowski supersymmetry.[6] There is no contradiction, however, because the set of preserved supersymmetries and the energy of a closed supertube are independent of the angular momentum. In fact, they are precisely the same as those of a supersymmetric collection of strings and D0-branes (see (2.5) and (2.21)). This follows from the supersymmetry algebra because both systems carry the same central charges: for a closed supertube, the total linear momentum (2.24) vanishes because in this case $y_i(\sigma)$ are periodic, and the net D2-brane charge vanishes because one of the D2-brane directions is compact.

[6]This should be contrasted with the case of AdS supersymmetry, for which the angular momentum *is* one of the central charges. Presumably, this is related to the supersymmetry of the giant gravitons [3].

The case of a supertube with an open cross-section which extends asymptotically to infinity[7] is more subtle, because in this case there is both a total linear momentum and a net D2-brane charge. Nevertheless, the supersymmetry algebra still implies the same bound on the energy and the same supersymmetry conditions because of a "cancellation" between the momentum and the D2-brane charge in the supersymmetry algebra. Indeed, preserved supersymmetries correspond to eigen-spinors ϵ of the matrix in (2.22) with zero eigen-value. Specifying to a supertube with cross-section extending asymptotically along a direction denoted by $\|$ eq. (2.22) becomes

$$\{Q,Q\} = P^0 + N\Gamma_T\Gamma_{||}\left(1-\Gamma_Z\right) + \Gamma_{TZ}\Gamma_\natural \mathcal{Z}_{F1} + \Gamma_T\Gamma_\natural \mathcal{Z}_{D0}\,, \tag{2.29}$$

where we have made use of the fact that $P_{||} = N$ (see (2.25)) and that $\mathcal{Z}^{D2}_{z||} = N$. Now it is clear that if we impose the conditions (2.5) on ϵ then the term in brackets originating from the linear momentum and the D2-brane charge automatically vanishes, and that in order for ϵ to have zero eigen-value we must have

$$P^0 = |\mathcal{Z}_{F1}| + |\mathcal{Z}_{D0}|\,. \tag{2.30}$$

This is essentially the integrated (in the Z-direction) version of eq. (2.21), and the usual arguments show that the right-hand side is a lower bound on the energy of any configuration with the same charges.

We now turn to the upper bound on the angular momentum. For simplicity, here we restrict ourselves to closed supertubes. We choose the parametrization such that $\vec{y}(\sigma) = \vec{y}(\sigma+2\pi)$, so that the angular momentum 2-form per period is

$$L_{ij} = N\int_0^{2\pi} d\sigma\,\left(y_i\,y'_j - y_j\,y'_i\right). \tag{2.31}$$

The total angular momentum J is defined as

$$J \equiv 2\pi\sqrt{\frac{1}{2}L_{ij}L^{ij}}\,, \tag{2.32}$$

and satisfies the bound[8]

$$J \leq N^{-1}\left|q_{F1}\,q_{D0}\right|. \tag{2.33}$$

This follows by TST-duality [13] from the results in [14], where an analogous bound on the angular momentum of a string was proven by expanding the periodic functions $y^i(\sigma)$ in harmonics on S^1. It also follows from the results in [14] that saturation of the bound is achieved if and only if: (*i*) the supertube cross-section is a curve in $\mathbb{E}^2$, so all but one of the skew-eigenvalues of L vanish, (*ii*) this curve is a circle, so the supertube is a perfect cylinder, and (*iii*) Π and B are constant, so the supertube carries homogeneous

[7]This must indeed be the case for a non-closed curve, since by charge conservation a D2-brane cannot have a boundary unless it ends on another appropriate brane.

[8]This bound was derived in [10] for a supertube with a circular cross-section.

string and D0-brane charge densities. For a supertube satisfying (i), that is, with cross-section contained within the (say) 12-plane, the other two conditions can be easily seen by parametrizing the cross-section in polar coordinates as

$$y^1 = R(\sigma)\,\sin\sigma\,, \qquad y^2 = R(\sigma)\,\cos\sigma\,, \tag{2.34}$$

where $R(\sigma)$ is a position-dependent radius. Note that we have made use of the residual reparametrization invariance left unfixed by the choice (2.2) to identify $\Phi = \sigma$, where Φ is the polar angle in the 12-plane. Integrating over a period we have the following chain of (in)equalities:

$$\begin{aligned} J &= 2\pi\, L_{12} = 2\pi\, N \int_0^{2\pi} d\sigma\, R^2(\sigma) \\ &\leq 2\pi\, N \int_0^{2\pi} d\sigma\, \left(R^2(\sigma) + R'^2(\sigma)\right) \\ &\leq 2\pi\, N \int_0^{2\pi} d\sigma\, |\vec{y}\,'(\sigma)|^2 \\ &= 2\pi\, N^{-1} \int_0^{2\pi} d\sigma\, |\Pi(\sigma)\, B(\sigma)| \\ &\leq N^{-1} \left(\int_0^{2\pi} d\sigma\, |\Pi(\sigma)|\right) \left(\int_0^{2\pi} d\sigma\, |B(\sigma)|\right) \\ &= N^{-1}\, |q_{F1}\, q_{D0}|\,, \end{aligned} \tag{2.35}$$

where we have made use of (2.13), (2.15), (2.23), (2.27), (2.32), and (2.34). Saturation of all the inequalities in (2.35) implies (ii) and (iii).

We finally turn to the linear momentum density. For a general 1/4-supersymmetric type-IIA configuration, the magnitude of $\vec{\mathcal{P}}_{\rm ph}$ is not actually given by (2.13), but instead is only bounded from above by the right-hand side of this equation; this observation is important for comparison with the supergravity description. The reason is that the magnitude of the string and D0-brane charge densities at any point along the cross-section of a supertube can be increased without increasing the momentum density and while preserving supersymmetry. This is because, as discussed exhaustively in [10] for a circular supertube and as we shall see in section 4 for a general one, there is no force between a supertube and strings (aligned along the Z-direction) or D0-branes (with charges of the same sign as those on the tube) placed at rest at arbitrary distances from each other; the combined system still preserves 1/4 supersymmetry. In particular, these extra strings and D0-branes can be superposed with the tube without being bound to it, hence increasing $|\Pi B|$ but leaving $|\vec{\mathcal{P}}|$ intact. Thus for a general combined system we conclude that

$$|\mathcal{P}|^2 \leq |\Pi B|\,. \tag{2.36}$$

3. Supermembrane analysis

We have just seen that the cross-section of a D2-brane supertube may have an arbitrary shape. We shall now show that this result can be understood from an M-theory perspective via the light-front-gauge supermembrane. This analysis provides a link between the DBI approach and the Matrix model approach, used in [7] to recover the supertube with circular cross-section and then generalized in [9] to a supertube with elliptical cross-section. The reason that the two approaches are related is that the D2-brane DBI action in a IIA vacuum background is equivalent to the action for the supermembrane action in a background spacetime of the form $\mathbb{E}^{(1,9)} \times S^1$; the former is obtained from the latter by dualization of the scalar field that gives the position of the membrane on the S^1 factor. But the supermembrane action in light-cone gauge is a supersymmetric gauge quantum mechanics (SGQM) model with the group $SDiff$ of area-preserving diffeomorphisms of the membrane as its gauge group, and this becomes a Matrix model when $SDiff$ is approximated by SU(N) [15]. Here we shall work directly with the $SDiff$ model, and take the decompactification limit.

In choosing the light-front gauge, one first chooses the metric

$$ds^2_{11} = dY^+ dY_- + \sum_{I=1}^{9} dY^I dY^I \,. \tag{3.1}$$

The physical worldvolume fields Y^I specify the position of the membrane in a 9-dimensional space $\mathbb{E}^9$. The bosonic light-front gauge hamiltonian density is

$$\mathcal{H} = \frac{1}{2}\left[\sum_I P_I^2 + \sum_{I<J} \{Y^I, Y^J\}^2\right], \tag{3.2}$$

where P_I is the momentum space variable conjugate to Y^I. The bracket $\{f, g\}$ of any two functions on the membrane is the Lie bracket

$$\{f, g\} \equiv \varepsilon^{ab} \partial_a f \partial_b g \,, \tag{3.3}$$

where σ^a ($a = 1, 2$) are the (arbitrary) membrane coordinates. Although the variables (Y^I, P_I) span an 18-dimensional space, the physical bosonic phase space is only 16-dimensional because of the Gauss-law constraint

$$\sum_I \left\{Y^I, P_I\right\} = 0 \,, \tag{3.4}$$

and the $SDiff$ gauge transformation that the constraint function generates.

We begin by separating the coordinates as $Y^I = (Z, Y^i)$, where Y^i are cartesian coordinates on $\mathbb{E}^8$, and their corresponding conjugate momenta as $P_I = (P_Z, P_i)$. The bosonic hamiltonian density is now

$$\mathcal{H} = \frac{1}{2}\left(\sum_i P_i^2 + P_Z^2 + \sum_{i<j} \{Y^i, Y^j\}^2 + \sum_i \{Y^i, Z\}^2\right), \tag{3.5}$$

and the Gauss-law constraint is

$$\{Y^i, P_i\} + \{Z, P_Z\} = 0 \,. \tag{3.6}$$

We will be interested in solutions that preserve 8 of the 16 supersymmetries of the D=11 supermembrane theory, and hence 1/4 of the supersymmetry of the M-theory vacuum.

We expect supersymmetric solutions of the membrane equations of motion to minimize the energy for fixed conserved charges, and we shall use this method to find them. Following [7] we use (3.6) to rewrite the hamiltonian density as

$$\mathcal{H} = \frac{1}{2}\sum_i \left(P_i - \{Z, Y^i\}\right)^2 + \frac{1}{2}P_Z^2 + \frac{1}{2}\sum_{i<j}\{Y^i, Y^j\}^2 + \varepsilon^{ab}\partial_a \left[Z \left(P_i \partial_b Y^i + P_Z \partial_b Z\right)\right]. \quad (3.7)$$

For fixed boundary conditions at any boundary of the membrane, the energy density is therefore minimized locally by configurations satisfying both

$$\{Y^i, Y^j\} = 0 \quad (3.8)$$

and

$$P_i = \{Z, Y^i\}, \qquad P_Z = 0. \quad (3.9)$$

For configurations with the tubular topology of the previous section the hamiltonian is

$$H = \frac{1}{2}\left[\int \{Z^2, Y^i\}\, dY^i\right], \quad (3.10)$$

where the integral is over the cross-sectional curve in $\mathbb{E}^8$ and the square brackets indicate an evaluation "at the ends" of the Z-direction.

To verify that configurations satisfying (3.8) and (3.9) indeed preserve the expected fraction of supersymmetry, we make use of the fact that the supersymmetry transformation of the 16-component worldvolume SO(9) spinor field S is

$$\delta S = \left[P_I \gamma^I + \frac{1}{2}\{Y^I, Y^J\}\gamma_{IJ}\right]\alpha + \beta, \quad (3.11)$$

where α and β are two 16-component constant SO(9) spinor parameters, and γ^I are the 16×16 SO(9) Dirac matrices. Clearly, all of the "β-supersymmetries" are broken, and the 16 components of S are the corresponding Nambu-Goldstone fermions. Setting β to zero and making use of the relations (3.8) and (3.9), the supersymmetry transformation becomes

$$\delta S = P_i \gamma_i \left(1 - \gamma_Z\right)\alpha, \quad (3.12)$$

where the three Dirac matrices γ^i form a reducible 16-dimensional representation of the Clifford algebra for SO(8). This vanishes for parameters α satisfying

$$\gamma_Z \alpha = \alpha, \quad (3.13)$$

which reduces the number of non-zero supersymmetry parameters to 8.

Let us now study some explicit solutions of eqs. (3.8) and (3.9). To make contact with [7] we first concentrate on a circular cross-section of radius R (which we initially allow to depend on the position along the axis of the tube) in the, say, 12-plane. Hence we choose membrane coordinates (z, φ) and set

$$Y^1 = R(z)\cos\varphi, \qquad Y^2 = R(z)\sin\varphi, \qquad Z = z. \quad (3.14)$$

It now follows from (3.8) that R must be constant, in which case Y^1, Y^2 and Z satisfy

$$\{Z, Y^1\} = -Y^2\,, \qquad \{Z, Y^2\} = Y^1\,, \qquad \{Y^1, Y^2\} = 0\,, \tag{3.15}$$

which implies that the angular momentum in the Z-direction is

$$L_Z = Y^1 P_2 - Y^2 P_1 = R^2\,. \tag{3.16}$$

If we now replace the Poisson bracket of functions on the membrane by $-i$ times the commutator of $N \times N$ hermitean matrices, then we recover the Matrix theory description of the supertube given in [7]. It is manifest from our derivation of this result that the large-N limit yields a tubular membrane with, in this case, a circular cross-section. It should be noted, however, that the identification of this 1/4-supersymmetric tubular membrane with the D2-brane supertube is not immediate because the latter lifts to a time-dependent M2-brane configuration in standard Minkowski coordinates [6]. Moreover, there exist 1/4-supersymmetric tubular solutions of the DBI equations for *non-constant* $R(z)$ [6], whereas supersymmetry forces constant R in the above Matrix model approach. Thus, eqs. (3.8) and (3.9) do not capture all possibilities for supersymmetric tubular D2-branes. However, the method does capture the possibility of an arbitrary cross-section. Indeed, choose membrane coordinates (z, σ) and set

$$Y^i = y^i(\sigma)\,, \qquad Z = z\,. \tag{3.17}$$

An argument analogous to that which led to (2.12) shows that we then have

$$P_i = y_i'\,. \tag{3.18}$$

It is now immediate to verify that (3.17) and (3.18) solve the supersymmetry equations (3.8) and (3.9).

4. Supergravity solution

The supergravity solution sourced by a supertube with arbitrary cross-section takes the form

$$\begin{aligned}
ds^2_{10} &= -U^{-1}V^{-1/2}\,(dT - A)^2 + U^{-1}V^{1/2}\,dZ^2 + V^{1/2}\,d\vec{Y}\cdot d\vec{Y}\,,\\
B_2 &= -U^{-1}\,(dT - A)\wedge dZ + dT\wedge dZ\,,\\
C_1 &= -V^{-1}\,(dT - A) + dT\,,\\
C_3 &= -U^{-1} dT\wedge dZ\wedge A\,, \qquad e^{\phi} = U^{-1/2}V^{3/4}\,,
\end{aligned} \tag{4.1}$$

where, as above, $\vec{Y}$ are cartesian coordinates on $\mathbb{E}^8$. $U(\vec{Y})$ and $V(\vec{Y})$ are harmonic functions on $\mathbb{E}^8$, and $A(\vec{Y})$ is a 1-form on $\mathbb{E}^8$ which must satisfy Maxwell's equation $d *_8 dA = 0$. B_2 and C_p are the Neveu-Schwarz and Ramond-Ramond potentials, respectively, with gauge-invariant field-strengths

$$H_3 = dB_2\,, \qquad F_2 = dC_1\,, \qquad G_4 = dC_3 - dB_2\wedge C_1\,. \tag{4.2}$$

Note for future reference that for the solution above

$$G_4 = U^{-1}V^{-1}(dT - A) \wedge dZ \wedge dA\,. \tag{4.3}$$

The solution (4.1) was actually presented in [10], but there only the choice of U, V and A that describes a supertube with circular cross-section was found. The generalization to an N-D2-brane tube with an *arbitrary* cross-section in $\mathbb{E}^8$ specified by $\vec{Y} = \vec{y}(\sigma)$ and carrying string and D0-brane charge densities $\Pi(\sigma)$ and $B(\sigma)$ is

$$U(\vec{Y}) = 1 + \frac{1}{6\Omega_7}\int d\sigma\, \frac{|\Pi(\sigma)|}{|\vec{Y} - \vec{y}(\sigma)|^6}\,, \tag{4.4}$$

$$V(\vec{Y}) = 1 + \frac{1}{6\Omega_7}\int d\sigma\, \frac{|B(\sigma)|}{|\vec{Y} - \vec{y}(\sigma)|^6}\,, \tag{4.5}$$

$$A(\vec{Y}) = \frac{N}{6\Omega_7}\int d\sigma\, \frac{y'_i(\sigma)}{|\vec{Y} - \vec{y}(\sigma)|^6}\, dY^i\,, \tag{4.6}$$

where Ω_q is the volume of a unit q-sphere. The solution (4.1) with these choices correctly reproduces all the features of the supertube. First, it preserves the same supersymmetries, since for any choice of U, V and A, the solution (4.1) is invariant under eight supersymmetries generated by Killing spinors of the form (in the obvious orthonormal frame for the metric)

$$\epsilon = U^{-1/4}V^{-1/8}\epsilon_0\,, \tag{4.7}$$

where ϵ_0 is a constant spinor which must satisfy precisely the constraints (2.5)[9] [10].

Second, it carries all the appropriate charges. To see this, we need to distinguish between closed and open cross-sections. Suppose first that the cross-section is closed. In this case we shall assume that it is contained in a compact region of $\mathbb{E}^8$ of finite size and that the charges (2.15) and momenta (2.24) and (2.27) are finite. Under these circumstances we have the following asymptotic behaviour for large $|\vec{Y}|$:

$$\begin{aligned} U(\vec{Y}) &\sim 1 + \frac{|q_{F1}|}{6\Omega_7|\vec{Y}|^6} + \cdots\,, \qquad V(\vec{Y}) \sim 1 + \frac{|q_{D0}|}{6\Omega_7|\vec{Y}|^6} + \cdots\,, \\ A(\vec{Y}) &\sim \frac{L_{ij}Y^j}{2\Omega_7|\vec{Y}|^8}\, dY^i + \cdots\,, \end{aligned} \tag{4.8}$$

where the dots stand for terms sub-leading in the limit under consideration and q_{F1}, q_{D0} and L_{ij} are defined as in (2.15) and (2.27). This shows that the metric is asymptotically flat for large $|\vec{Y}|$. We see from the contributions of U and V to the field-strengths H_3 and F_2 that the solution carries string charge q_{F1} and D0-brane charge per unit length in the Z-direction q_{D0}. Evaluation of the appropriate ADM integrals shows that it also carries an energy per unit length in the Z-direction

$$\mathcal{H} = |q_{F1}| + |q_{D0}|\,, \tag{4.9}$$

as well as angular momentum L_{ij} in the ij-plane. Note that there is no linear momentum, as expected for a closed curve. The field-strength (4.3) sourced by the D2-brane takes the

[9] With $\mathrm{sgn}(E) = \mathrm{sgn}(B) = 1$; these signs are reversed by taking $T \to -T$ and/or $Z \to -Z$.

asymptotic form

$$G_4 \sim d\left[\frac{L_{ij}Y^j}{2\Omega_7|\vec{Y}|^8}\, dT\wedge dZ\wedge dY^i\right] + \cdots . \tag{4.10}$$

Since the integral of $*G_4$ over any 6-sphere at infinity vanishes, the solution carries no net D2-brane charge, as expected for a closed supertube. However, as explained in [10], the fact that G_4 does not vanish implies that the D2-brane dipole (and higher) moments are non-zero.

Consider now an open cross-section. For simplicity we assume that outside some compact region the curve becomes a straight line along the, say, Y^1-direction, and that the densities per unit area $\Pi_{\rm ph}(\sigma)$ and $\mathrm{B}_{\rm ph}(\sigma)$ become constants Π_0 and B_0. Under these circumstances we have the following asymptotic expansions

$$U \sim 1+\frac{|\Pi_0|}{5\Omega_6|\vec{Y}_\perp|^5}+\cdots , \qquad V \sim 1+\frac{|B_0|}{5\Omega_6|\vec{Y}_\perp|^5}+\cdots , \qquad A \sim \frac{N}{5\Omega_6|\vec{Y}_\perp|^5}\, dY^1+\cdots , \tag{4.11}$$

for large $|\vec{Y}_\perp|$, where

$$\vec{Y}_\perp = (Y^2,\ldots,Y^8) \tag{4.12}$$

is the position in the transverse directions. These expansions show that, at large distances, the solution describes N infinite D2-branes that extend asymptotically along the ZY^1-plane, with strings (along the Z-direction) and D0-branes bound to them. Indeed, the solution is asymptotically flat at tranverse infinity, that is, for large $|\vec{Y}_\perp|$. From the asymptotic forms of H_3 and F_2 we see that it carries string and D0-brane charge densities Π_0 and B_0, respectively. The energy density is $|\Pi_0|+|B_0|$. We also see from the asymptotic behaviour of the "$dTdY^1$-term" in the metric that, unlike in the closed case, there are now N units of net linear momentum in the 1-direction per unit area. Similarly, now there are N units of net D2-brane charge, since asymptotically we have

$$G_4 \sim d\left[\frac{N}{5\Omega_6|\vec{Y}_\perp|^5}\, dT\wedge dZ\wedge dY^1\right] + \cdots . \tag{4.13}$$

This is precisely the long-distance field-strength associated to N D2-branes oriented along the ZY^1-plane.

The third feature which allows the solution with the choices (4.4)–(4.6) to be identified with the supertube is that (by construction) it is singular at and only at the location of the tube $\vec{Y}=\vec{y}(\sigma)$. In fact, the behaviour of the solution in the region close to the singularity will allow us to reproduce the bound (2.36). Let $\vec{Y}_0 = \vec{y}(\sigma_0)$ be a point on the tube, $\vec{v} = \vec{y}\,'(\sigma_0)$ the tangent vector and $\Pi_0 = \Pi(\sigma_0)$ and $B_0 = B(\sigma_0)$ the string and D0-brane densities at that point. By performing an appropriate translation and rotation if necessary we assume that $\vec{Y}_0 = 0$ and that $\vec{v}$ lies along the 1-direction. It is now straightforward to see that in the limit $\vec{Y}\to\vec{Y}_0$ we have

$$\begin{aligned} U &\sim 1+\frac{|\Pi_0|}{5\Omega_6|\vec{v}||\vec{Y}_\perp|^5}+\cdots , \qquad V \sim 1+\frac{|B_0|}{5\Omega_6|\vec{v}||\vec{Y}_\perp|^5}+\cdots , \\ A &\sim \frac{|\vec{\mathcal{P}}|}{5\Omega_6|\vec{v}||\vec{Y}_\perp|^5}dY^1+\cdots , \end{aligned} \tag{4.14}$$

where $|\vec{\mathcal{P}}| = N|\vec{v}|$, with $\vec{Y}_\perp$ as in (4.12) and where the dots stand for $\vec{Y}_\perp$-dependent subleading terms. We see that in this limit the tube behaves as an infinite planar D2/F1/D0-bound state extending along the ZY^1-plane which carries a momentum density $\vec{\mathcal{P}}$ along the 1-direction. The bound on this density arises now as follows. Consider the vector field $\ell \equiv \partial/\partial Y^1$, which becomes tangent to the curve as $\vec{Y} \to \vec{Y}_0$. Its norm squared is

$$|\ell|^2 = U^{-1}V^{-1/2}\left(UV - A_1^2\right). \tag{4.15}$$

This is always positive sufficiently far away from the tube, and if the bound (2.36) is satisfied it remains spacelike everywhere. However, if the bound is violated then ℓ becomes timelike sufficiently close to the point $\vec{Y}_0$. To see this we note that, as $\vec{Y} \to \vec{Y}_0$, (4.15) becomes

$$|\ell|^2 = \left(5\Omega_6|\vec{Y}_\perp|^5\right)^{-1/2} |\Pi_0|^{-1} |B_0|^{-1/2} |\vec{v}|^{-2} \left[(|\Pi_0 B_0| - |\mathcal{P}|^2) + \cdots\right], \tag{4.16}$$

where the dots stand for non-negative terms which vanish in this limit. Thus $|\ell|^2$ becomes negative sufficiently close to $\vec{Y}_0$ if and only if $|\mathcal{P}|^2 > |\Pi_0 B_0|$. If this happens at every point along the cross-section then curves almost tangent and sufficiently close to the supertube are timelike. For a closed cross-section this leads to a global violation of causality since these become closed timelike curves. This case was analyzed in detail in [10] for a circular cross-section, where it was shown that regions with timelike ℓ also lead to another pathology: the appearance of ghost degrees of freedom on appropriate brane probes. The reason is that the coefficient of the kinetic energy of certain fields on the probe is proportional to $|\ell|^2$. Since this is a local instability, it will still occur even if the bound (2.36) is violated only locally and no closed timelike curves are present. Hence, it is the requirement of stability of brane probes that leads to the bound (2.36) in the supergravity description of supertubes with arbitrary cross-sections.

5. Absence of tachyons

In this section we compute the vacuum energy of the open strings stretched between tangent planes of the supertube and show that no tachyons are present. We follow the light-cone gauge boundary state formalism of [16] closely, although in that paper the formalism was developed explicitly only for type-IIB D-branes. Here we provide the slight modification necessary to treat the type-IIA D-branes.

5.1 Generalities

The boundary states constructed in [16] in the light-cone-gauge formalism describe "$(p+1)$-instantons" with euclidean worldvolume. Since each such $(p+1)$-instanton is related to an ordinary Dp-brane with lorentzian worldvolume by a double Wick rotation we shall just use the term Dp-brane in the following.

In the light-cone description the spacetime coordinates are divided into the light-cone coordinates X^+, X^- and the transverse coordinates X^I ($I = 1, \ldots, 8$). We shall further separate the latter into coordinates X^i ($i = 1, \ldots, p+1$) parallel to the Dp-brane and coordinates X^n ($n = p+2, \ldots, 8$) transverse to it.

JHEP03(2002)016

The boundary state $|B\rangle$ corresponding to an infinite planar Dp-brane or anti-Dp-brane is completely determined by the orientation of the brane in spacetime and by the world-volume BI 2-form field-strength F_{ij}. This information is encoded in an O(8) "rotation" constructed as follows.

Consider first a Dp-brane extended along the directions $1, \ldots, p$. Define the O(8) matrix

$$M_{IJ} = \begin{pmatrix} M_{ij} & \\ & \mathbb{I}_{7-p} \end{pmatrix}, \tag{5.1}$$

where $\mathbb{I}_q$ is the $(q \times q)$-dimensional identity matrix and

$$M_{ij} = -\left[(1-F)(1+F)^{-1}\right]_{ij}. \tag{5.2}$$

Note that in the type-IIB theory p is odd and hence $\det M = 1$; this is the case considered explicitly in [16]. We shall instead concentrate on the type-IIA theory, for which p is even and hence $\det M = -1$. In this case it is convenient to write M as

$$M = \mathcal{D} \cdot \tilde{M}, \tag{5.3}$$

where

$$\mathcal{D} \equiv \begin{pmatrix} -\mathbb{I}_{p+1} & \\ & \mathbb{I}_{7-p} \end{pmatrix}, \qquad \tilde{M} \equiv \mathcal{D} \cdot M. \tag{5.4}$$

Note that $\mathcal{D}^2 = 1$ and $\det \tilde{M} = 1$.

The rotation M_{IJ} gives rise to two different elements in the spinor representation differing by a sign. In particular, if we write

$$\tilde{M}_{IJ} = \exp\left(\frac{1}{2}\Omega_{KL}\Sigma^{KL}\right)_{IJ}, \tag{5.5}$$

where Σ^{KL} are the generators of SO(8) in the vector representation and $\Omega_{KL} = -\Omega_{LK}$ are constants, then these two elements are

$$M_{ab} = \pm\left(\mathcal{D} \cdot \tilde{M}\right)_{ab}, \tag{5.6}$$

where[10] $a, b = 1, \ldots, 8$, γ^I are SO(8) Dirac gamma-matrices and

$$\mathcal{D}_{ab} = \left(\gamma^{p+2}\gamma^{p+3}\cdots\gamma^8\right)_{ab}, \qquad \tilde{M}_{ab} = \exp\left(\frac{1}{4}\Omega_{IJ}\gamma^{IJ}\right)_{ab}. \tag{5.7}$$

The two possible choices of sign in (5.6) correspond to the possibilities, for fixed orientation and BI field-strength, of a D-brane or an anti-D-brane.

Consider now a Dp-brane with an arbitrary orientation obtained from the previous one by an SO(8) rotation $m(\phi)$, specified by some angles collectively denoted by ϕ. Then the corresponding matrix $M(\phi)$ is

$$M(\phi) = m^{-1}(\phi) \cdot M \cdot m(\phi). \tag{5.8}$$

[10] Although in the type-IIA theory the left-moving and the right-moving spinors have opposite chiralities, we shall not explicitly distinguish between chiral and anti-chiral spinor indices, that is, between dotted and undotted indices.

In particular, when $F = 0$, a rotation of angle $\phi = \pi$ on a 2-plane with one direction parallel to the brane and another direction orthogonal to it leaves M_{IJ} invariant but reverses the sign of M_{ab}, and hence it transforms a D-brane into an anti-D-brane and viceversa. This is the generalization to branes of any dimension of the familiar fact that reversing the orientation of a string tranforms it into an anti-string.

The boundary state is completely specified once M is known, and takes the form

$$|B\rangle = \delta_{\perp}^{(p+1)} \, R(\tilde{M}) \exp \sum_{n>0} \left(\frac{1}{n} \mathcal{D}_{IJ} \alpha_{-n}^{I} \tilde{\alpha}_{-n}^{J} - i \mathcal{D}_{ab} S_{-n}^{a} \tilde{S}_{-n}^{b} \right) |B_0\rangle \,, \tag{5.9}$$

where the delta function above has support on the worldvolume of the brane (see appendix A for details), and the zero-mode factor is

$$|B_0\rangle = C \left(M_{IJ} |I\rangle |J\rangle + i M_{ab} |a\rangle |b\rangle \right) . \tag{5.10}$$

The normalization constant C is given in terms of the BI field-strength on the D-brane as

$$C(F) = \sqrt{\det(1+F)} \,. \tag{5.11}$$

Finally, the operator $R(\tilde{M})$ is the representation of the SO(8)-rotation $\tilde{M}$ on non-zero modes:

$$R(\tilde{M}) = \exp \sum_{n>0} \left(\frac{1}{n} T_{IJ}^{(\alpha)} \alpha_{-n}^{I} \alpha_{n}^{J} + T_{ab}^{(S)} S_{-n}^{a} S_{n}^{b} \right) , \tag{5.12}$$

where

$$T_{IJ}^{(\alpha)} = \frac{1}{2} \Omega_{KL} \Sigma_{IJ}^{KL} \,, \qquad T_{ab}^{(S)} = \frac{1}{4} \Omega_{KL} \gamma_{ab}^{KL} \,. \tag{5.13}$$

This satisfies the group property $R(\tilde{M}_1) R(\tilde{M}_2) = R(\tilde{M}_1 \tilde{M}_2)$.

The vacuum energy of a system consisting of two D-branes is due (to the lowest order in the string coupling constant) to the exchange of closed strings between them, and is given by

$$Z = \int_0^{\infty} \frac{dt}{2p^+} \langle B_2 | e^{-(P_{cl}^- - p^-)t} | B_1 \rangle \,, \tag{5.14}$$

where $|B_k\rangle$ $(k = 1, 2)$ is the boundary state for the k-th brane, characterized by a matrix M_k. Z factorizes as

$$Z = Z_0 \, Z_{\rm osc} \,, \tag{5.15}$$

where $Z_{\rm osc}$ depends only on the relative rotation between the branes

$$M_{\rm rel} \equiv M_2^T \cdot M_1 \,, \tag{5.16}$$

and

$$Z_0 = C(F_1) C(F_2) \left({\rm Tr}_v \; M_{\rm rel} - {\rm Tr}_s \; M_{\rm rel} \right) , \tag{5.17}$$

where ${\rm Tr}_v$ and ${\rm Tr}_s$ are the traces in the vector and the spinor representation, respectively. Note that, except for the product of the normalization factors in Z_0, the vacuum energy depends only on the relative rotation $M_{\rm rel}$, which in turn depends only on the relative rotation between the SO(8) parts of M_1 and M_2, that is, on

$$\tilde{M}_{\rm rel} \equiv \tilde{M}_2^T \cdot \tilde{M}_1 = M_{\rm rel} \,. \tag{5.18}$$

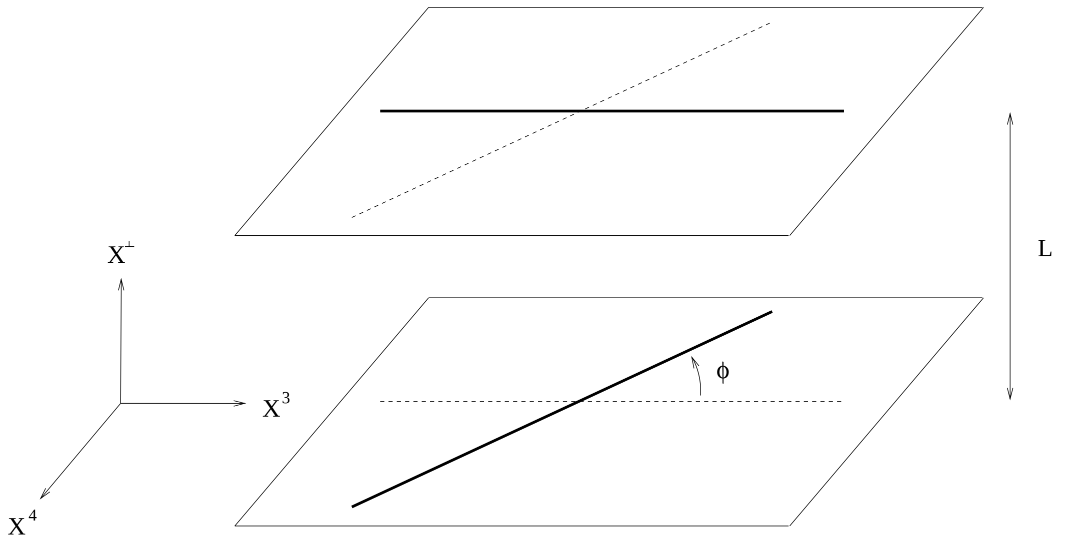

Figure 2: Setup of D2-branes tangent to the supertube. Both branes share the 12-directions. The thick lines represent their projections on the 6-dimensional space orthogonal to the 12-plane, and both lie along the 34-plane. The "first" ("second") brane is that on the plane above (below).

5.2 Vacuum energy

As explained in the Introduction, in order to gain some understanding about the possible presence of tachyons in the spectrum of open strings stretched between arbitrary points of a supertube, one can approximate the worldvolume at each of these points by that of an infinite planar D2-brane carrying string and D0-brane charges. In this section we will show that the vacuum energy of two such branes vanishes exactly.

It is clear that the two D2-branes of interest will share the time direction, which in our euclidean formulation we may take to be X^1. They will also share one spatial direction, X^2 say, corresponding to the axis of the supertube (the Z-direction). The projection of each brane on the remaining 6-dimensional space orthogonal to the 12-plane is a line in this space (see figure 2). For a supertube with a generic cross-section these two lines will determine a 2-plane, the 34-plane say, and will form some angle ϕ with each other, but will not intersect. Instead they will be separated some distance L along the remaining 4 overall orthogonal directions $X^\perp$. The particular case $\phi = \pi$ corresponds to a parallel D2/anti-D2 pair.

In order to construct the boundary states describing each brane, we assume without loss of generality that both branes are initially aligned along the 3-direction, and that we then rotate the (say) second brane by an angle ϕ. Thus before the rotation both branes extend along the 123-directions. Since they carry string charge along the 2-direction and D0-brane charge, the BI 2-forms F_k $(k = 1, 2)$ on their worldvolumes are

$$F_k = E_k \, dX^1 \wedge dX^2 + B_k \, dX^2 \wedge dX^3 \,, \tag{5.19}$$

where for the moment we keep the values of the electric and magnetic fields arbitrary. The matrices M^k_{IJ} then take the form (5.1) with

$$M^k_{ij} = \frac{1}{C^2(F_k)} \begin{pmatrix} -1+E_k^2-B_k^2 & 2E_k & -2E_kB_k \\ -2E_k & -1+E_k^2+B_k^2 & 2B_k \\ -2E_kB_k & -2B_k & -1-E_k^2+B_k^2 \end{pmatrix}, \tag{5.20}$$

where the normalization factor is

$$C(F_k) = \sqrt{1+E_k^2+B_k^2}\,. \tag{5.21}$$

The corresponding spinorial matrix (5.6) for either brane is

$$M^k_{ab} = \frac{1}{C(F_k)}\,\gamma^{12\cdots 8}\cdot\left(\gamma^{123}+E_k\gamma^3+B_k\gamma^1\right)_{ab}\,. \tag{5.22}$$

Note that we have kept the "plus" sign in (5.6) for both branes because we are assuming that we start with two parallel branes of the same type, that is, either with two D-branes or with two anti-D-branes. As mentioned above, the D-brane/anti-D-brane case is achieved by setting $\phi = \pi$.

The matrix $M_2(\phi)$ describing the second brane after the rotation is now obtained from M_2 by conjugation with the rotation matrix $m(\phi)$, as in (5.8). In the present case $m(\phi)$ takes the form

$$m(\phi)_{IJ} = \exp\left(\phi\,\Sigma^{34}\right)_{IJ} = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \cos\phi & -\sin\phi & \\ & & \sin\phi & \cos\phi & \\ & & & & \mathbb{I}_4 \end{pmatrix} \tag{5.23}$$

in the vector representation, and

$$m(\phi)_{ab} = \exp\left(\frac{\phi}{2}\gamma^{34}\right)_{ab} = \begin{pmatrix} e^{i\phi/2}\,\mathbb{I}_4 & \\ & e^{-i\phi/2}\,\mathbb{I}_4 \end{pmatrix} \tag{5.24}$$

in the spinor representation.

Now we are ready to evaluate the vacuum energy (5.14) explicitly. The zero-mode factor (5.17) is given by

$$Z_0(F_1,F_2) = \frac{\lambda}{\sqrt{(1+E_1^2+B_1^2)(1+E_2^2+B_2^2)}} - 8\left[(1+E_1E_2)\cos\phi + B_1B_2\right], \tag{5.25}$$

where

$$\begin{aligned}\lambda = 2\,\big[&3+E_1^2+E_2^2+4E_1E_2+3E_1^2E_2^2+4B_1^2B_2^2+2B_1^2(1+E_2^2)+2B_2^2(1+E_1^2)+ \\ &+4B_1B_2(1+E_1E_2)\cos\phi+(1+E_1^2)(1+E_2^2)\cos 2\phi\big]\,.\end{aligned} \tag{5.26}$$

In the supersymmetric case $Z_0 = 0$. To see this, recall that for a supertube with arbitrary shape we have $E = \pm 1$ and the only restriction on B is that it does not change sign along the curve. The first condition becomes $E_1 = E_2 = \pm i$ after euclideanization, in which case

$$Z_0 = 8\left(|B_1B_2| - B_1B_2\right) = 0\,, \tag{5.27}$$

which vanishes since B_1 and B_2 have equal signs by virtue of the second condition. Furthermore, setting $E_k = i + \varepsilon_k$ and expanding Z_0 for small ε_k one finds that the first terms are of order ε^2, as expected from 1/4-supersymmetry. Note that Z_0 might also vanish for other values of E_k, B_k and ϕ, but we shall not explore this possibility here.

The vanishing of Z_0 implies that the vacuum energy Z vanishes as well *provided* that $Z_{\rm osc}$ is finite. This is not guaranteed *a priori*, however, because if a tachyonic mode is present in the spectrum of open strings stretched between the branes then $Z_{\rm osc}$ will diverge. Thus we need to evaluate $Z_{\rm osc}$ before concluding that $Z = 0$ and hence that there is no force between the branes.[11] This can be done by redefining the oscillators in (5.12):

$$\begin{aligned} \alpha_n^I &\longrightarrow U_{IJ}\alpha_n^J\,, \qquad & \alpha_{-n}^I &\longrightarrow U_{IJ}^\dagger \alpha_{-n}^J\,, \\ S_n^a &\longrightarrow V_{ab} S_n^b\,, \qquad & S_{-n}^a &\longrightarrow V_{ab}^\dagger S_{-n}^b\,, \end{aligned} \tag{5.28}$$

where the unitary matrices U and V are defined so that they diagonalize the T-generators (5.13) of the relative rotation $M_{\rm rel} = \tilde{M}_{\rm rel}$, that is,

$$\begin{aligned} U^\dagger T^{(\alpha)} U &= \begin{pmatrix} i\beta_1 &&&& \\ & -i\beta_1 &&& \\ && i\beta_2 && \\ &&& -i\beta_2 & \\ &&&& \mathbb{O}_4 \end{pmatrix}, \\ V^\dagger T^{(S)} V &= \begin{pmatrix} i\beta_+ \mathbb{I}_2 &&& \\ & i\beta_- \mathbb{I}_2 && \\ && -i\beta_+ \mathbb{I}_2 & \\ &&& -i\beta_- \mathbb{I}_2 \end{pmatrix}, \end{aligned} \tag{5.29}$$

where

$$\beta_\pm = \frac{\beta_1 \pm \beta_2}{2}\,. \tag{5.30}$$

Since $M_{\rm rel}$ is effectively a rotation in the 1234-directions it is characterized by only two angles β_1 and β_2. These can be extracted from the relations

$$\begin{aligned} \cos\left(\frac{\beta_1}{2}\right)\cos\left(\frac{\beta_2}{2}\right) &= \frac{1}{8}\,{\rm Tr}_s\, M_{\rm rel}\,, \\ \cos\beta_1 + \cos\beta_2 &= \frac{1}{16}\left[({\rm Tr}_s\, M_{\rm rel})^2 - 2\,{\rm Tr}_s\, M_{\rm rel}^2 - 16\right]. \end{aligned} \tag{5.31}$$

Note that the traces are in the spinor representation, since the vector representation of M does not distinguish between brane and anti-brane. A straightforward calculation shows that for our supersymmetric configurations

$${\rm Tr}_s\, M_{\rm rel} = 8\ {\rm sgn}(B_1 B_2)\,, \qquad {\rm Tr}_s\, M_{\rm rel}^2 = 8\,, \tag{5.32}$$

and hence that $\beta_1 = \beta_2 = 0$.

[11] An additional motivation to evaluate $Z_{\rm osc}$ is that (althought this is not our main interest here) the entire spectrum of the open strings stretched between the branes can be read off from it.

The evaluation of $Z_{\rm osc}$ now proceeds as in [16], with the result

$$Z_{\rm osc} = \int_0^\infty dt \frac{\prod_{n=1}^\infty (1-q^{2n}e^{i\beta_+})^2(1-q^{2n}e^{-i\beta_+})^2(1-q^{2n}e^{i\beta_-})^2(1-q^{2n}e^{-i\beta_-})^2}{\prod_{n=1}^\infty (1-q^{2n})^4(1-q^{2n}e^{i\beta_1})(1-q^{2n}e^{-i\beta_1})(1-q^{2n}e^{i\beta_2})(1-q^{2n}e^{-i\beta_2})} \times \\ \times \mathcal{P}(t,\phi,L)\,, \tag{5.33}$$

where $q=e^{-\pi t}$ and the factor

$$\mathcal{P}(t,\phi,L) = \begin{cases} V_2\,(2\pi^2 t)^{-3}\,|\sin\phi|^{-1}\,\exp\left(-L^2/2\pi t\right)\,, & \text{if } \phi \neq 0 \mod \pi\,, \\ V_3\,(2\pi^2 t)^{-7/2}\,\exp\left(-L^2/2\pi t\right)\,, & \text{if } \phi = 0 \mod \pi\,, \end{cases} \tag{5.34}$$

arises essentially from the delta-functions in (5.9) (see appendix A). In particular, the constants V_2 and V_3 are the (infinite) volumes of the branes in the 12- and 123-directions, respectively. In view of the periodicity of $Z_{\rm osc}$, we assume that β_1, β_2, β_+ and β_- all lie within the interval $[0,2\pi)$.

The result can be rewritten in a compact form in terms of Jacobi θ-functions and the Dedekind η-function:

$$Z_{\rm osc} = \int_0^\infty dt\, \frac{\sin\left(\frac{\beta_1}{2}\right)\sin\left(\frac{\beta_2}{2}\right)}{4\sin^2\left(\frac{\beta_+}{2}\right)\sin^2\left(\frac{\beta_-}{2}\right)} \frac{\theta_1^2\left(\frac{\beta_+}{2\pi}\mid it\right)\theta_1^2\left(\frac{\beta_-}{2\pi}\mid it\right)}{\eta^6(it)\,\theta_1\left(\frac{\beta_1}{2\pi}\mid it\right)\theta_1\left(\frac{\beta_2}{2\pi}\mid it\right)} \mathcal{P}(t,\phi,L)\,. \tag{5.35}$$

The presence or absence of an open string tachyon is most clearly exhibited in the short cylinder limit, $t\to 0$. To study this we perform a Jacobi transformation $t \to t' = 1/t$. This gives

$$Z_{\rm osc} = \int_0^\infty \frac{dt'}{t'}\, \frac{\sin\left(\frac{\beta_1}{2}\right)\sin\left(\frac{\beta_2}{2}\right)}{4\sin^2\left(\frac{\beta_+}{2}\right)\sin^2\left(\frac{\beta_-}{2}\right)} \frac{\theta_1^2\left(-it'\frac{\beta_+}{2\pi}\mid it'\right)\theta_1^2\left(-it'\frac{\beta_-}{2\pi}\mid it'\right)}{\eta^6(it')\,\theta_1\left(-it'\frac{\beta_1}{2\pi}\mid it'\right)\theta_1\left(-it'\frac{\beta_2}{2\pi}\mid it'\right)} \times \\ \times \mathcal{P}\left(\frac{1}{t'},\phi,L\right). \tag{5.36}$$

Since the argument of θ_1 is now imaginary, the behaviour of the relevant part of the integrand in the short cylinder limit, $t'\to\infty$, is given by[12]

$$e^{-t'\left(L^2-\pi|\beta_1-\beta_2|\right)/2\pi} + \mathcal{O}(e^{-\pi t'})\,. \tag{5.37}$$

Thus we see that generically $Z_{\rm osc}$ diverges for separations $L^2 \leq \pi|\beta_1-\beta_2|$. As mentioned above, this divergence signals the appearance of a tachyonic instability. Supersymmetric configurations are free of this instability, as we wanted to see, since in this case $\beta_1=\beta_2=0$. Note that, in fact, there is no tachyonic instability in any brane configuration with $\beta_1=\beta_2$, although generically there will still be a force between the branes unless $Z_0=0$. Again, we shall not explore these more general possibilities here.

In conclusion, the vacuum energy of two D2-branes tangent to the supertube vanishes exactly. This means that there are no tachyonic instabilites in the system and that there is no force between the branes (at the lowest order in the string coupling constant).

[12] $Z_{\rm osc}$ might appear to diverge (to vanish) when β_1 and/or β_2 (β_+ and/or β_-) vanish because of the sine factors in (5.36). This is not the case, however, as is manifest from (5.33) or as can be seen directly from (5.36) by noting that $\lim_{\nu\to 0}\theta_1(\nu|\tau)/\sin(\pi\nu) = 2\eta^3(\tau)$ for any finite τ. Thus the formula (5.37) gives the correct asymptotic behaviour of the integrand for any value of β_1 and β_2.

6. Discussion

We have generalized the 1/4-supersymmetric D2-brane supertube with circular cross-section [6], to one for which the cross-section is an *arbitrary* curve in $\mathbb{E}^8$. This obviously includes the system of a parallel 1/4-supersymmetric D2-brane and anti-D2-brane of [9] as a special case. In many respects, this result is counter-intuitive. While it is not difficult to imagine how a circular tube may be supported from collapse by angular momentum, it is less obvious how this is possible when the cross-section is non-circular and non-planar: one would imagine that this would imply a clash between the need for angular momentum to prevent collapse and the need for time-independence of the energy density to preserve supersymmetry. Of course this problem already arose for the elliptical supertube found in [9] but that was found in the context of the Matrix model and, as we have pointed out, the implications of results in Matrix Theory for supertube configurations are not immediately clear. Nevertheless, it was the results of [9] that motivated us to re-examine the assumption of circular symmetry for supertubes in the DBI context and, on finding that any cross-section is possible, to re-examine the reasons underlying the stability of the supertube.

We identified three sources of instability in the Introduction. The most obvious one is the potential local instability due to D2-brane tension and we have now dealt fairly exhaustively with that. In particular, we have seen how the D2-brane charge in the spacetime supersymmetry algebra can be "cancelled" by the linear momentum generated by the BI fields; this for example allows a D2-brane extending to infinity (and hence carrying a net charge) to have an arbitrary cross-section while preserving supersymmetry. This simple mechanism of "brane-charge cancellation by momentum" may have other applications.

A second potential instability is due to long-range supergravity forces. This was dealt with for a supertube with a circular cross-section in [10]. Here we have generalized that analysis to an arbitrary cross-section. A main consequence of this generalization is that the bound on the total angular momentum of [10] becomes a bound on the local linear momentum density. Violation of the bound in [10] was shown to lead to the appearance of closed timelike curves and ghost-induced instabilities on brane probes. Here we have seen that although local violations of the bound on the momentum density do not necessarily imply the presence of closed timelike curves, they do induce ghost instabilities on brane probes. Bound-violating solutions are consequently unstable, whereas bound-respecting solutions are stable at the supergravity level.

Finally, there is a potential source of instability due to tachyon condensation, and one of the principal purposes of this paper has been to demonstrate that no tachyons appear in the spectrum of open strings ending on a supertube. We have accomplished slightly less than this since existing methods apply only to planar branes. Specifically, we have shown that no tachyons appear in the spectrum of open strings connecting any two tangent planes of the supertube. This includes the case of strings connecting a D2-brane with an anti-D2-brane, provided that the DBI fields are those required for the preservation of 1/4 supersymmetry.

JHEP03(2002)016

Note added in proof. While this paper was being type-written we received [17], where some results which partially overlap with those of our section 5 are obtained by different methods.

Acknowledgments

It is a pleasure to thank Michael B. Green for very helpful discussions, and Iosif Bena for correspondence. D.M. is supported by a PPARC fellowship. S.N. is supported by the British Federation of Women Graduates and the Cambridge Philosophical Society.

A. Calculation of the $\mathcal{P}$-factor in $Z_{\rm osc}$

In this appendix we briefly explain how the $\mathcal{P}$-factor in $Z_{\rm osc}$ arises. For clarity, we explicitly put hats on operators.

Consider the general case of two Dp-branes which extend along p common directions (collectively denoted as $X_{||}$), whose projections on the 34-plane are as in figure 2, and which lie at the positions $x^{\perp}_{\it 1}$ and $x^{\perp}_{\it 2}$ in the remaining $6-p$ overall transverse directions. At the end we will specialize to the case of interest to us for which $p=2$, $X_{||}=(X^1,X^2)$ and $|x^{\perp}_{\it 2}-x^{\perp}_{\it 1}|=L$.

The $\mathcal{P}$-factor in $Z_{\rm osc}$ is

$$\mathcal{P}=\langle p=0|\,\delta^{\perp}_{\it 2}\,q^{\hat{P}^2/2}\,\delta^{\perp}_{\it 1}\,|p=0\rangle\equiv\mathcal{P}_{||}\,\mathcal{P}_{3-4}\,\mathcal{P}_{\perp}\,, \tag{A.1}$$

where $q=e^{-\pi t}$,

$$\begin{aligned}\mathcal{P}_{||}&=\langle p_{||}{=}0|p_{||}{=}0\rangle\,,\\ \mathcal{P}_{3-4}&=\langle p_3{=}p_4{=}0|\delta(\hat{X}'_3)\,q^{\hat{P}_3^2/2}\,\delta(\hat{X}_3)|p_3{=}p_4{=}0\rangle\,,\\ \mathcal{P}_{\perp}&=\langle p_{\perp}{=}0|\,\delta(\hat{X}^{\perp}-x^{\perp}_{\it 2})\,q^{\hat{P}_{\perp}^2/2}\,\delta(\hat{X}^{\perp}-x^{\perp}_{\it 1})|p_{\perp}{=}0\rangle\,,\end{aligned} \tag{A.2}$$

and

$$\hat{X}'_3=\cos\phi\,\hat{X}_3+\sin\phi\,\hat{X}_4\,. \tag{A.3}$$

The first factor is just the p-dimensional (infinite) volume of the directions common to both branes, $\mathcal{P}_{||}=V_p$. The second one is easily evaluated by first introducing an integral representation of the delta function,

$$\begin{aligned}\mathcal{P}_{3-4}&=\int\frac{dk}{2\pi}\,\langle p_3{=}p_4{=}0|\,\delta(\hat{X}'_3)\,q^{\hat{P}_3^2/2}\,e^{ik\hat{X}_3}|p_3{=}p_4{=}0\rangle\\ &=\int\frac{dk}{\sqrt{2\pi}}\,q^{k^2/2}\,\langle p_3{=}p_4{=}0|\,\delta(\cos\phi\,\hat{X}_3+\sin\phi\,\hat{X}_4)\,|p_3{=}k,\,p_4{=}0\rangle\,,\end{aligned} \tag{A.4}$$

and then introducing a resolution of the identity in position space,

$$\mathcal{P}_{3-4}=\int\frac{dk}{2\pi}\,q^{k^2/2}\int dx_3\int dx_4\,e^{ikx_3}\,\delta(\cos\phi\,x_3+\sin\phi\,x_4)\,. \tag{A.5}$$

Now we need to distinguish two cases. If $\sin\phi \neq 0$, then integrating first over x_4 we get

$$\mathcal{P}_{3-4} = \frac{1}{|\sin\phi|}\int \frac{dk}{2\pi}\, q^{k^2/2} \int dx_3\, e^{ikx_3} = \frac{1}{|\sin\phi|}\,. \tag{A.6}$$

If $\sin\phi = 0$ then $|\cos\phi| = 1$ and we get

$$\mathcal{P}_{3-4} = \ell \int \frac{dk}{2\pi}\, q^{k^2/2} \int dx_3\, e^{ikx_3}\, \delta(x_3) = \ell\, \left(2\pi^2 t\right)^{-1/2}\,, \tag{A.7}$$

where ℓ is the (infinite) length of the 4-direction.

A similar computation shows that

$$\mathcal{P}_{\perp} = \left(2\pi^2 t\right)^{-d^{\perp}/2} \exp\left(-\frac{\left|x_1^{\perp} - x_2^{\perp}\right|^2}{2\pi t}\right), \tag{A.8}$$

where $d^{\perp} = 6 - p$ is the number of overall transverse directions.

In conclusion

$$\mathcal{P} = \begin{cases} V_p\, |\sin\phi|^{-1}\, \left(2\pi^2 t\right)^{-\frac{d^{\perp}}{2}} \exp\left(-\frac{\left|x_1^{\perp}-x_2^{\perp}\right|^2}{2\pi t}\right) & \text{if } \phi \neq 0 \bmod \pi\ , \\ V_{p+1}\, \left(2\pi^2 t\right)^{-\frac{d^{\perp}+1}{2}} \exp\left(-\frac{\left|x_1^{\perp}-x_2^{\perp}\right|^2}{2\pi t}\right) & \text{if } \phi = 0 \bmod \pi\ , \end{cases} \tag{A.9}$$

where $V_{p+1} = V_p \ell$.

Specializing to the case of interest to us yields (5.34).

References

[1] R. Emparan, *Born-Infeld strings tunneling to D-branes*, *Phys. Lett.* **B 423** (1998) 71 [hep-th/9711106].

[2] R.C. Myers, *Dielectric-branes*, *J. High Energy Phys.* **12** (1999) 022 [hep-th/9910053].

[3] J. McGreevy, L. Susskind and N. Toumbas, *Invasion of the giant gravitons from anti-de Sitter space*, *J. High Energy Phys.* **06** (2000) 008 [hep-th/0003075].

[4] J. Polchinski and M.J. Strassler, *The string dual of a confining four-dimensional gauge theory*, hep-th/0003136.

[5] C.V. Johnson, A.W. Peet and J. Polchinski, *Gauge theory and the excision of repulson singularities*, *Phys. Rev.* **D 61** (2000) 086001 [hep-th/9911161].

[6] D. Mateos and P.K. Townsend, *Supertubes*, *Phys. Rev. Lett.* **87** (2001) 011602 [hep-th/0103030].

[7] D. Bak and K.-M. Lee, *Noncommutative supersymmetric tubes*, *Phys. Lett.* **B 509** (2001) 168 [hep-th/0103148].

[8] T. Harmark and K.G. Savvidy, *Ramond-Ramond field radiation from rotating ellipsoidal membranes*, *Nucl. Phys.* **B 585** (2000) 567 [hep-th/0002157].

[9] D.-S. Bak and A. Karch, *Supersymmetric brane-antibrane configurations*, *Nucl. Phys.* **B 626** (2002) 165 [hep-th/0110039].

JHEP03(2002)016

[10] R. Emparan, D. Mateos and P.K. Townsend, *Supergravity supertubes*, *J. High Energy Phys.* **07** (2001) 011 [hep-th/0106012].

[11] A. Sen, *Tachyon condensation on the brane antibrane system*, *J. High Energy Phys.* **08** (1998) 012 [hep-th/9805170].

[12] E. Bergshoeff, R. Kallosh, T. Ortin and G. Papadopoulos, *Kappa-symmetry, supersymmetry and intersecting branes*, *Nucl. Phys.* **B 502** (1997) 149 [hep-th/9705040].

[13] D. Mateos, S. Ng and P.K. Townsend, *Supercurves*, to appear.

[14] O. Lunin and S.D. Mathur, *Metric of the multiply wound rotating string*, *Nucl. Phys.* **B 610** (2001) 49 [hep-th/0105136].

[15] B. de Wit, J. Hoppe and H. Nicolai, *On the quantum mechanics of supermembranes*, *Nucl. Phys.* **B 305** (1988) 545.

[16] M.B. Green and M. Gutperle, *Light-cone supersymmetry and D-branes*, *Nucl. Phys.* **B 476** (1996) 484 [hep-th/9604091].

[17] D.-S. Bak and N. Ohta, *Supersymmetric D2 anti-D2 strings*, *Phys. Lett.* **B 527** (2002) 131 [hep-th/0112034].